Rules of Exponents

$x^3 \bullet x^2 =$

$x \bullet x \bullet x \bullet x \bullet x = x^5$

$(x^3)^2 =$

$(x \bullet x \bullet x) \bullet (x \bullet x \bullet x) = x^6$

$$\frac{x^7}{x^4} = \frac{\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot x\cdot x\cdot x}{\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{x}} = x^3$$

$$\frac{y^2x^{-3}z^5}{y^{-4}x^6z^7} = \frac{y^2\boxed{x^{-3}}z^5}{\boxed{y^{-4}}x^6z^7} = \frac{y^4y^2z^5}{x^3x^6z^7} =$$

$$\frac{y\cdot y\cdot y\cdot y\cdot y\cdot y\cdot \cancel{z}\cdot\cancel{z}\cdot\cancel{z}\cdot\cancel{z}\cdot\cancel{z}}{x\cdot x\cdot x\cdot x\cdot x\cdot x\cdot x\cdot x\cdot x\cdot \cancel{z}\cdot\cancel{z}\cdot\cancel{z}\cdot\cancel{z}\cdot\cancel{z}\cdot z\cdot z}$$

$$\frac{y^6}{x^9\cdot z^2}$$

$a^{\frac{3}{4}} = \sqrt[4]{a^3}$ *or* $\left(\sqrt[4]{a}\right)^3$

1. Rewrite the following **rational exponents** in $\sqrt{\textit{radical form}}$ and simplify if possible. *(provide an approximate answer when possible).*

a. $x^{\frac{4}{5}}$

$\left(\sqrt[5]{x}\right)^4 = \boxed{\sqrt[5]{x^4}}$

b. $48^{\frac{2}{3}} = \sqrt[3]{48^2}$

$= \left(\sqrt[3]{48}\right)^2$

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48^(2/3)
        13.207709
3×√(48²)
        13.207709
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OR $4\sqrt[3]{36}$

$\boxed{= \sqrt[3]{48^2}}$

decimal approximation: ≈ 13.21

c. $(32x)^{\frac{3}{4}} = \left(\sqrt[4]{32x}\right)^3$

32 → 8, 4 → 4, 2, 2, 2 → 2, 2, 2, 2, 2x

$= \left(2^1\sqrt[4]{2x}\right)^3$

$= 2^3\cdot\sqrt[4]{(2x)^3}$

$\boxed{8\sqrt[4]{8x^3}}$

d. $\frac{27^{-\frac{4}{3}}}{1} = \frac{1}{27^{4/3}}$

$= \frac{1}{\left(\sqrt[3]{27}\right)^4} = \frac{1}{(3)^4} = \frac{1}{3\cdot3\cdot3\cdot3}$

27 → 9, 3 → 3, 3, 3

$\boxed{\frac{1}{81}}$

2. First, simplify the **rational exponents** using rules of exponents and then rewrite the final answer in $\sqrt{\textit{radical form}}$.

a. $x^{\frac{1}{5}}\cdot x^{\frac{3}{5}} = x^{4/5}$

$\frac{1}{5}+\frac{3}{5}=\frac{4}{5}$

$\left(\sqrt[5]{x}\right)^4$ OR $\boxed{\sqrt[5]{x^4}}$

b. $\frac{a^{\frac{5}{3}}}{a^{\frac{1}{3}}}$

$\frac{5}{3}-\frac{1}{3}=\frac{4}{3}$

$= a^{4/3}$

$\left(\sqrt[3]{a}\right)^4$ OR $\boxed{\sqrt[3]{a^4}}$

c. $\frac{m^{\frac{2}{3}}\cdot m^{-\frac{1}{2}}}{m^{\frac{1}{6}}}$

$\frac{2\cdot2}{2\cdot3}+\frac{-1\cdot3}{2\cdot3}$

$\frac{4}{6}+\frac{-3}{6}=\frac{1}{6}$

$\frac{M^{1/6}}{M^{1/6}}$

$M^0 = 1$

$\boxed{1}$

3. First, simplify the **rational exponents** using rules of exponents and then rewrite the final answer in $\sqrt{radical\ form}$.

a. $\left(b^{\frac{1}{4}}\right)^4$

$\frac{1}{4}\cdot\frac{4}{1}=\frac{4}{4}=1$

b^1

$\boxed{b}$

b. $\left(p^{\frac{3}{4}}\right)^{\frac{2}{5}}$

$\frac{3}{4}\cdot\frac{2}{5}=\frac{6}{20}=\frac{3}{10}$

$p^{\frac{3}{10}}$

$\left(\sqrt[10]{p}\right)^3$ or $\boxed{\sqrt[10]{p^3}}$

c. $\left(\dfrac{w^{\frac{1}{6}}}{w^{-\frac{2}{3}}}\right)^2$

$\frac{1}{6}-\frac{-2\cdot 2}{3\cdot 2}$

$\frac{1}{6}+\frac{4}{6}=\frac{5}{6}$

$=\left(w^{5/6}\right)^2$

$\frac{5}{6}\cdot\frac{2}{1}=\frac{10}{6}=\frac{5}{3}$

$=w^{5/3}=\sqrt[3]{w^5}=$

$\left(\sqrt[3]{w}\right)^5$

www ww

$\boxed{=w\sqrt[3]{w^2}}$

4. Using your calculator approximate the following to the nearest hundredth.

a. $32^{\frac{3}{5}}$

32^(3/5)
8

$\boxed{8}$

b. $\sqrt[4]{54}$

4 ×√(54)
2.710806011

$\boxed{\approx 2.71}$

c. $\left(12^{\frac{1}{4}}\right)^{\frac{2}{3}}$

(12^(1/4))^(2/3)
1.513085749
12^(2/12)
1.513085749

$\boxed{\approx 1.51}$

d. $\sqrt[4]{8^{\frac{2}{3}}}$

4 ×√(8^(2/3))
1.414213562

$\boxed{\approx 1.41}$

e. $\left(\frac{243}{32}\right)^{-\frac{2}{5}}$

(243/32)^(-2/5)
.4444444444

$\boxed{=\frac{4}{9}\approx 0.44}$

6. Rewrite the following $\sqrt{radicals}$ using **rational exponents.**

a. $\sqrt[9]{p^3}=p^{\frac{3}{9}}$

$=p^{1/3}$

$\boxed{p^{1/3}}$

b. $\left(\sqrt[4]{5a}\right)^3=(5a)^{3/4}$

$5^{3/4}\cdot a^{3/4}=\boxed{(5a)^{3/4}}$

c. $\sqrt[5]{(2x)^{10}}=(2x)^{\frac{10}{5}}$

$=(2x)^2$

$=4x^2$

$\boxed{4x^2}$

d. $\sqrt[3]{27x^4y^6}=\left(27x^4y^6\right)^{\frac{1}{3}}$

$=27^{1/3}\cdot x^{4/3}\cdot y^{6/3}$

$\boxed{3x^{4/3}y^2}$

6. Rewrite the following using fractional exponents and simplify when possible:

a. $\sqrt[6]{x^4}+\sqrt[3]{x^6}$

$=x^{\frac{4}{6}}+x^{\frac{3}{6}}$

$=x^{2/3}+x^{1/2}$

$\boxed{x^{\frac{2}{3}}+x^{\frac{1}{2}}}$

b. $\sqrt[6]{h^2}\cdot\sqrt[3]{h}$

$=\left(h^{\frac{2}{6}}\right)\cdot\left(h^{\frac{1}{3}}\right)$

$=h^{1/3}\cdot h^{1/3}$

$\boxed{\frac{1}{3}+\frac{1}{3}=\frac{2}{3}}$

$\boxed{h^{2/3}}$

c. $\dfrac{\sqrt[2]{x^1}}{x^3}=\dfrac{x^{\frac{1}{2}}}{x^3}=\dfrac{1}{x^{5/2}}$

$\boxed{\frac{2\cdot 3}{2\cdot 1}-\frac{1}{2} \quad \frac{6}{2}-\frac{1}{2}=\frac{5}{2}}$

$x^{-5/2}$ or $\boxed{\dfrac{1}{x^{5/2}}}$

d. $\dfrac{\sqrt[3]{p^5}}{p\cdot\sqrt[3]{p}}=\dfrac{p^{5/3}}{p^1\cdot p^{1/3}}$

$\boxed{1+\frac{1}{3}=\frac{3}{3}+\frac{1}{3}=\frac{4}{3}}$

$\dfrac{p^{5/3}}{p^{4/3}}=p^{1/3}$

$\boxed{\frac{5}{3}-\frac{4}{3}=\frac{1}{3}}$

$\boxed{p^{1/3}}$